Bias, Variance, and MSE of Maximum Likelihood and Bayesian Estimators of the Mean of a Univariate Normal Distribution

Case 1: Bayesian Estimator has larger bias away from the prior mean, lower variance everywhere, lower MSE near the prior mean

- $n = 10$
- $\gamma_{\text{prior}} = 0$
- $\xi_{\text{prior}} = 1$
- $\xi = 1$

With these settings, the MLE is $\bar{X}$ and the Bayesian estimator is $\frac{10}{11} \bar{X}$.
Case 2: Location where Bayesian Estimator has lower MSE depends on the prior mean.

- $n = 10$
- $\gamma_{\text{prior}} = 2.5$
- $\xi_{\text{prior}} = 1$
- $\xi = 1$

With these settings, the MLE is $\bar{X}$ and the Bayesian estimator is $\frac{10}{11}\bar{X} + \frac{1}{11}2.5$.
Case 3: Range of values where Bayesian Estimator has lower MSE depends on the relative sizes of $\xi$ and $\zeta_{prior}$.

- $n = 10$
- $\gamma_{prior} = 0$
- $\zeta_{prior} = 0.25$
- $\xi = 1$

With these settings, the MLE is $\bar{X}$ and the Bayesian estimator is $\frac{10}{10.25} \bar{X}$.

![Graph showing Bias, Variance, and MSE for different estimators]