Suppose v and w are n-dimensional vectors;

$$v = (v_1, v_2, \dots, v_n)^T \quad \text{and} \quad w = (w_1, w_2, \dots, w_n)^T.$$

- The dot product of v and w is

$$v \cdot w = v^T w = v_1 w_1 + v_2 w_2 + \dots + v_n w_n = \sum_{i=1}^{n} v_i w_i$$

- The norm of v is its length:

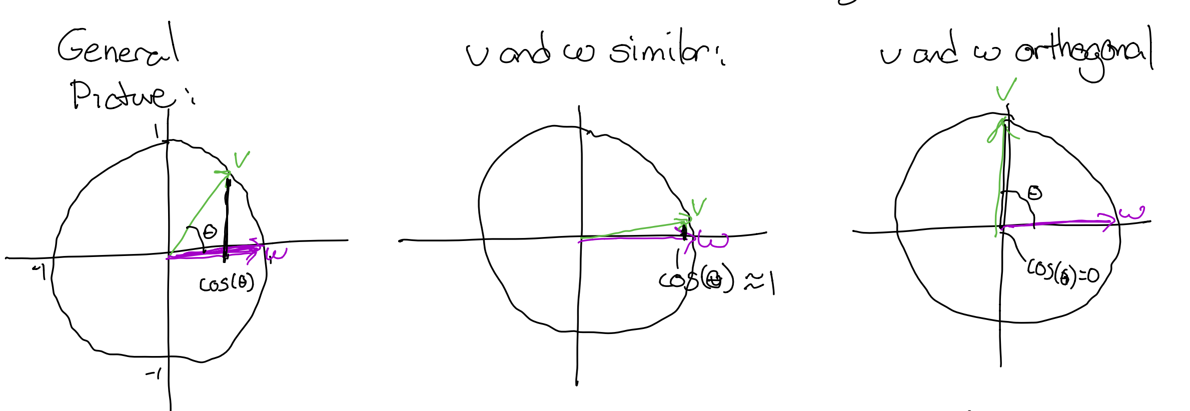
$$\|v\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2} = \sqrt{v \cdot v}$$

Theorem: Let θ be the angle between vectors v and w.

Then $v \cdot w = \|v\| \cdot \|w\| \cdot \cos(\theta)$

Interpretation:

- $\|v\|$ is length of v
- $\|w\|$ is length of w.
- $\cos(\theta)$ is a measure of similarity between v and w.



Main take away: $v \cdot w$ has built in a measure of similarity between v and w.

$\rightarrow$ $v \cdot w \approx 0 \Rightarrow$ v and w point in orthogonal directions, and/or they are short

$\rightarrow$ $v \cdot w$ large $\Rightarrow$ v & w point in a similar direction and are long.