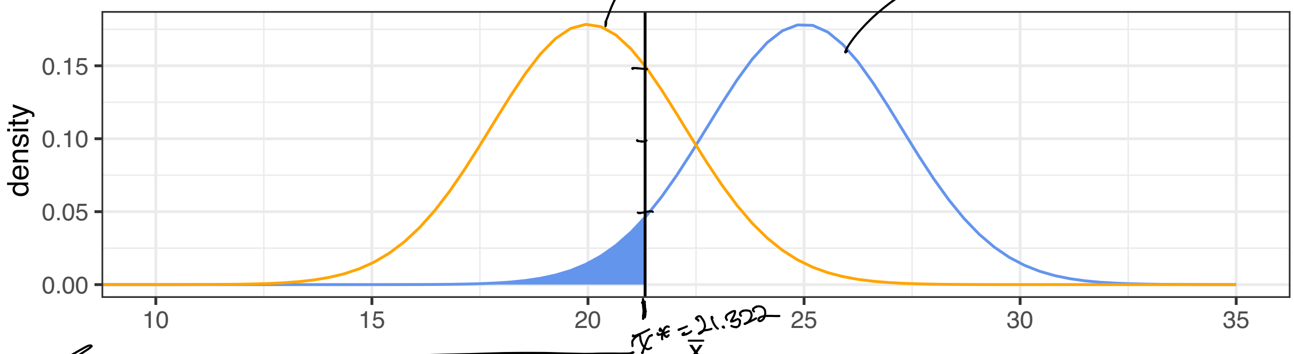
Likelihood Ratios

- Data Model: $X_1, \dots, X_5 \overset{\text{i.i.d.}}{\sim} \text{Normal}(\theta, 5^2)$
- Let's consider a test of the hypotheses $H_0 : \theta = 25$ vs. $H_A : \theta = 20$
- If H_0 is correct, then $\bar{X} \sim \text{Normal}(25, 5^2/5)$. If H_A is correct, then $\bar{X} \sim \text{Normal}(20, 5^2/5)$
- Two ways to think of the specification of the rejection region for the likelihood ratio test:

$f_{\bar X}(\bar x | \theta_A)$ $\quad$ $f_{\bar X}(\bar x | \theta_0)$

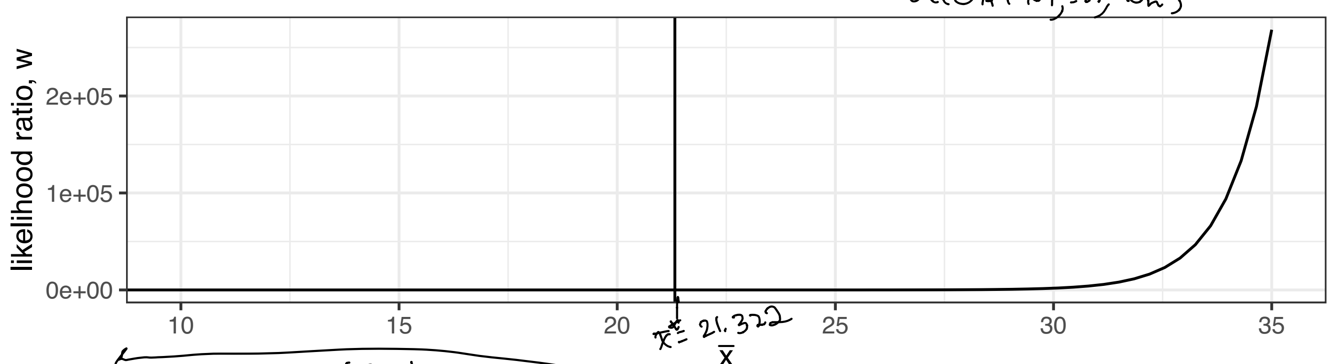
1. Reject H_0 if $\bar{x} \leq 21.322$



$\bar x^* = 21.322$

reject H_0 if $f_{\bar X}(\bar x|\theta_A) > 3 \cdot f_{\bar X}(\bar x|\theta_0)$

2. Reject H_0 if $W \leq w^*$ where W is the likelihood ratio statistic $W = \dfrac{\mathcal{L}(\theta_0 | x_1, \dots, x_n)}{\mathcal{L}(\theta_A | x_1, \dots, x_n)}$



$\bar x^* = 21.322$

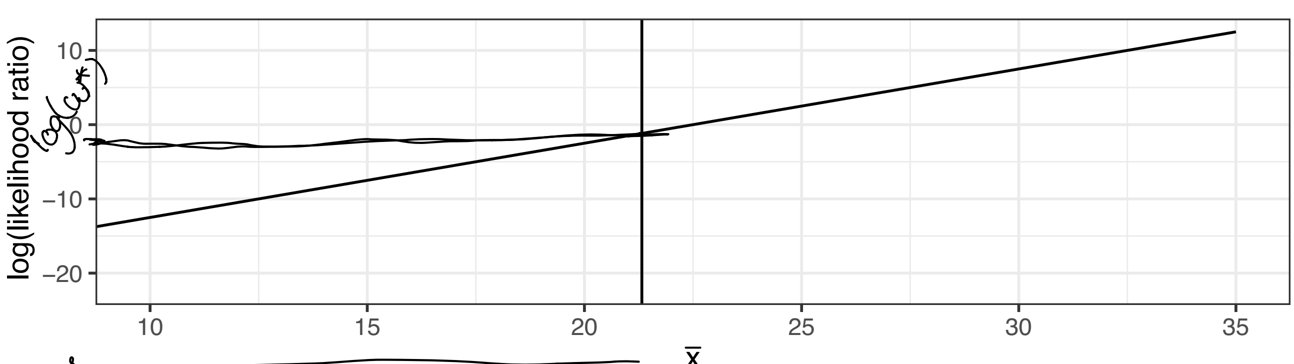
reject

The scale of the likelihood ratio is challenging; let's look at the log:

reject if $w \le w^*$

reject if $\log(w) \le \log(w^*)$

$\log(w^*)$



Reject H_0 if $\dfrac{f_{X_1,\dots,X_n|\Theta}(x_1,\dots,x_n|\theta_0)}{f_{X_1,\dots,X_n|\Theta}(x_1,\dots,x_n|\theta_A)} < w^*$

Reject H_0 if $w^* \cdot f_{X_1,\dots,X_n}(x_1,\dots,x_n|\theta_A) - f_{X_1,\dots,X_n|\Theta}(x_1,\dots,x_n|\theta_0) > 0$

$$\underline{w}^* f_{X_1,\dots,X_n}(x_1,\dots,x_n|\theta_A) > f_{X_1,\dots,X_n|\Theta}(x_1,\dots,x_n|\theta_0)$$