Truth

| Decision | $H_0$ true | $H_0$ false |
|---|---|---|
| Reject $H_0$ | Type I Error | |
| Fail to Reject $H_0$ | | Type II Error |

$P(\text{Type II Error} \mid H_0 \text{ false}) = \beta$

$P(\text{Fail to reject } H_0 \mid H_0 \text{ false})$

$P(\text{p-value} > \alpha \mid H_0 \text{ false})$

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Ex: Suppose $\alpha = 0.3$. Find $\beta$.

$P(\text{p-value} > 0.3 \mid H_0 \text{ false})$

$= P(\text{p-value} = 0.7 \mid H_0 \text{ false}) + P(\text{p-value} = 0.5 \mid H_0 \text{ false}) + P(\text{p-value} = 1 \mid H_0 \text{ false})$

$= 0.1 + 0.4 + 0.1 = 0.6$

Suppose $\alpha = 0.5$. Find $\beta$.

$$\beta = P(\text{p-value} > 0.5 \mid H_0 \text{ false})$$

$$= P(\text{p-value} = 0.7 \mid H_0 \text{ false}) + P(\text{p-value} = 1 \mid H_0 \text{ false})$$

$$= 0.1 + 0.1 = 0.2$$

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- In general, a small significance level (corresponding to small $P(\text{Type I Error} \mid H_0 \text{ true})$) corresponds to a high $P(\text{Type II Error} \mid H_0 \text{ false})$

- On the other hand, increase $\alpha \iff$ higher $P(\text{Type I Error} \mid H_0 \text{ true})$ $\iff$ lower $P(\text{Type II Error} \mid H_0 \text{ false})$

Power of a test:

$P(\text{Reject } H_0 \mid H_0 \text{ false})$

$= 1 - P(\text{Fail to reject } H_0 \mid H_0 \text{ false})$

$= 1 - P(\text{Type II Error} \mid H_0 \text{ false})$

$= 1 - \beta$

Suppose $\alpha = 0.3$ and $H_0$ false. Find the power of the test.

$\text{Power} = 1 - \beta = 1 - 0.6 = 0.4$

Suppose $\alpha = 0.5$ and $H_0$ false. Find power:

$\text{Power} = 1 - \beta = 1 - 0.2 = \underline{0.8}$

with this test at a significance level of 0.5, P(correctly reject $H_0 \mid H_0$ false) is 0.8.