		Truth	
		H_0 true	H_0 false
Decision	Reject H_0	Type I Error	:)
	Fail to Reject H_0	:)	Type II Error

The <u>size</u> of a test is

$$P(\text{Type I Error} \mid H_0 \text{ true})$$

By definition, $P(\text{Type I Error} \mid H_0 \text{ true}) \le \alpha$

<u>size</u> $\le$ <u>significance level</u>

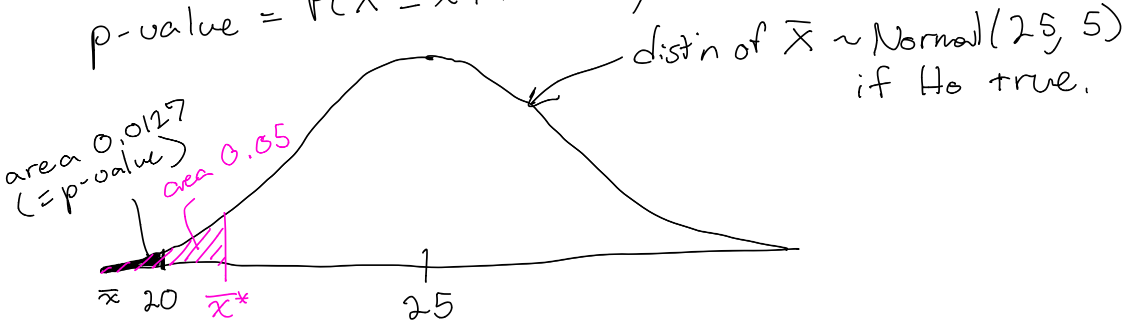
Continue example from lab 15:

$X_1, \ldots, X_5 \sim \text{Normal}(\theta, 5^2)$

$H_0: \theta = 25$

$H_A: \theta = 10$

p-value $= P(\bar{X} \le \bar{x} \mid H_0 \text{ true})$



critical value: the value of the test statistic so the corresponding p-value would be exactly the significance level α.

2 approaches to decision about reject or fail to reject H_0:

1) Compare p-value to α, reject if p-value $< \alpha$

p-value $= 0.017 < 0.05$, so reject H_0.

2) Compare $\bar{x}$ (the observed value of the test statistic) to $\bar{x}^*$, a critical value.

Reject H_0 if $\bar{x}$ is "more extreme" than the critical value.

For these 2 approaches to be equivalent,

p-value $< \alpha$ if and only if $\bar{x}$ more extreme than $\bar{x}^*$

- Set up for critical value:

$\bar{x}^*$ is the value of test statistic so that p-value = significance level of test.

$P(\bar{X}$ "more extreme than critical value" $| H_0$ true$) = \alpha$

- If $\bar{x}$ more extreme than $\bar{x}^*$, then

$P(\bar{X}$ "more extreme than" $\bar{x} | H_0$ true$) < \alpha$

Size of test $= P(\text{Type I Error} \mid H_0 \text{ true})$

$= P(\text{Reject } H_0 \mid H_0 \text{ true})$

$= P(\bar{X}$ more extreme than $\bar{x}^* | H_0$ true$)$

$= \alpha$

$=$ significance level of the test.

For example of discrete distribution,

suppose $\alpha = 0.1$.

What is $P(\text{Type I Error} \mid H_0 \text{ true})$?

$$
\begin{aligned}
P(\text{Type I Error} \mid H_0 \text{ true}) &= P(\text{p-value} \le \alpha \mid H_0 \text{ true}) \\
&= P(\text{p-value} \le 0.1 \mid H_0 \text{ true}) \\
&= 0
\end{aligned}
$$

size of test $= 0$, but significance level $= 0.1$.

With discrete distributions, could get size < significance level.

Suppose $\alpha = 0.3$.

$$
\begin{aligned}
P(\text{Type I Error} \mid H_0 \text{ true}) &= P(\text{p-value} \le 0.3 \mid H_0 \text{ true}) \\
&= P(\text{p-value} = 0.2 \mid H_0 \text{ true}) \\
&= 0.2
\end{aligned}
$$