

Hypothesis Testing - Likelihood Ratio Warm-Up

Adapted from Rice Exercise 9.4

Suppose X is a random variable that can take one of the values in the set $\{1, 2, 3, 4\}$. X follows one of two distributions, with probability mass functions given in the table below. We want to conduct a test of the hypotheses

H_0 : the first distribution is correct

H_A : the second distribution is correct

$$\text{Likelihood ratio: } W = \frac{L(\theta_0 | x)}{L(\theta_A | x)} = \frac{f_X(x | \text{null})}{f_X(x | \text{alternative})}$$

x	$f_X(x)$ for H_0	$f_X(x)$ for H_A	Likelihood Ratio
1	0.2	0.1	$0.2/0.1 = 2$
2	0.3	0.4	$0.3/0.4 = 0.75$
3	0.3	0.1	$0.3/0.1 = 3$
4	0.2	0.4	$0.2/0.4 = 0.5$

1. Fill in the table above with values of the likelihood ratio corresponding to each possible value of x .

2. Suppose we take a sample and we observe $x = 2$. What is the p-value for the likelihood ratio test?

If $x=2$, then $w = 0.75$.

$$\begin{aligned} \text{p-value} &= P(W \leq 0.75 | H_0 \text{ is correct}) = P(W=0.75 | H_0 \text{ correct}) + P(W=0.5 | H_0 \text{ correct}) \\ &= 0.3 + 0.2 = 0.5 \end{aligned}$$

3. In this example, what are the possible p-values for the likelihood ratio test? (Find the p-values for the test if we observe $x = 1$, $x = 3$, and $x = 4$)

$$\text{If } x=1 \text{ then } w = 2. \quad \text{p-value} = 0.2 + 0.3 + 0.2 = 0.7$$

$$\text{If } x=3 \text{ then } w = 3 \quad \text{p-value} = 0.2 + 0.3 + 0.3 + 0.2 = 1$$

$$\text{If } x=4 \text{ then } w = 0.5 \quad \text{p-value} = 0.2$$