Large-sample confidence interval example: Seedlings (Poisson Model)

- Observe $X_1, \ldots, X_n$; X_i is the number of seedlings in quadrat number i.
- Data Model: $X_i|\Lambda = \lambda \overset{\text{i.i.d.}}{\sim} \text{Poisson}(\lambda)$
- We have seen that the maximum likelihood estimate is $\hat{\Lambda}^{MLE} = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$
- A large-sample approximate confidence interval is $[\bar{X} - z(1-\frac{\alpha}{2})\sqrt{\frac{\bar{x}}{n}}, \bar{X} - z(\frac{\alpha}{2})\sqrt{\frac{\bar{x}}{n}}]$

Here are the results for a 95% CI:

```
x_bar <- mean(seedlings$new_1993)
x_bar
```

```
## [1] 0.7333333
```

```
n <- nrow(seedlings)

ci_lower <- x_bar - qnorm(0.975) * sqrt(x_bar / n)
ci_lower
```

```
## [1] 0.5166512
```

```
ci_upper <- x_bar - qnorm(0.025) * sqrt(x_bar / n)
ci_upper
```

```
## [1] 0.9500155
```

Compare to our data:

```
x_grid <- 0:5
pmfs <- bind_rows(
  data.frame(x = x_grid, pmf = dpois(x_grid, lambda = x_bar), param = "MLE"),
  data.frame(x = x_grid, pmf = dpois(x_grid, lambda = ci_lower), param = "CI Lower"),
  data.frame(x = x_grid, pmf = dpois(x_grid, lambda = ci_upper), param = "CI Upper")
)

ggplot() +
  geom_bar(data = seedlings, mapping = aes(x = new_1993, y = ..prop..)) +
  geom_point(data = pmfs, mapping = aes(x = x, y = pmf, color = param, shape = param)) +
  theme_bw()
```