$X_1, \dots, X_n \overset{iid}{\sim}$ Normal (μ, σ^2)

Confidence interval for μ:

Overall structure:

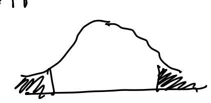
1. $\underline{\dfrac{\bar{X} - \mu}{S/\sqrt{n}}} \sim \underline{t_{n-1}}$

The quantity $\dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ is an example of a <u>pivotal quantity</u>: a function of observable data $(X_1, \dots, X_n)$ and unknown parameters (μ) whose distribution does not depend on unknown parameters.

- $\dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ depends on $X_1, \dots, X$ and μ

- Distribution (t_{n-1}) doesn't depend on unknown parameters.

2. Write down a relevant probability statement involving the pivotal quantity.



$$P\left(t_{n-1}\left(\tfrac{\alpha}{2}\right) \le \frac{\bar{X} - \mu}{S/\sqrt{n}} \le t_{n-1}\left(1 - \tfrac{\alpha}{2}\right)\right) = 1 - \alpha$$

3. Do algebra: rearrange to get a form like

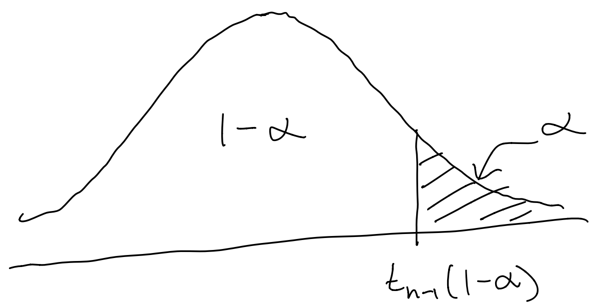
$$P(A \le \mu \le B) = 1 - \alpha$$

$[A, B]$ is a $(1-\alpha) * 100\%$ C.I. for μ.

Could also derive a one-sided C.I.

① $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$

② $P\left(\frac{\bar{X}-\mu}{S/\sqrt{n}} \le t_{n-1}(1-\alpha)\right) = 1-\alpha$



③ $P\left(-\mu \le -\bar{X} + t_{n-1}(1-\alpha)\frac{S}{\sqrt{n}}\right) = 1-\alpha$

$\Rightarrow P\left(\mu \ge \bar{X} - t_{n-1}(1-\alpha)\frac{S}{\sqrt{n}}\right) = 1-\alpha$

So a $(1-\alpha)100\%$ CI for μ is:

$$\left[\bar{X} - t_{n-1}(1-\alpha)\frac{S}{\sqrt{n}},\ \infty\right)$$