Confidence Interval for Variance of a normal distribution

Suppose $X_1, \ldots, X_n \overset{\text{i.i.d.}}{\sim} \text{Normal}(\mu, \sigma^2)$ with both μ and σ^2 unknown.

(a) We have previously stated that $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$, where $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$. Explain in a sentence or two why this is a pivotal quantity for σ^2.

(b) Use the pivotal quantity from part (a) to find a 2-sided confidence interval for σ^2. At this point we are not working with a sample of observed data so your interval endpoints should be random variables.

Hint: Note that if $\frac{1}{5} < \frac{1}{4} < \frac{1}{3}$, you can take the reciprocal of all three terms if you also reverse the direction of the inequalities: $5 > 4 > 3$.