Large-Sample Normal Approximation to the Posterior

First Observaton: for large n, the prior doesn't matter

Poisson Model

- Observe $X_1, \dots, X_n$; X_i is the number of seedlings in quadrat number i.
- Data Model: $X_i | \Lambda = \lambda \overset{\text{i.i.d.}}{\sim} \text{Poisson}(\lambda)$
- Suppose we use a Gamma prior for Λ
  - Example: $\Lambda \sim \text{Gamma}(1, 0.01)$ is fairly non-informative
- Think of the posterior pdf in terms of contributions from the prior and the likelihood

$$f_{\Lambda | X_1, \dots, X_n}(\lambda | x_1, \dots, x_n) \propto f_\Lambda(\lambda) \cdot f_{X_1, \dots, X_n | \Lambda}(x_1, \dots, x_n | \lambda)$$

$$\propto c \cdot f_\Lambda(\lambda) \cdot \prod_{i=1}^{n} f_{X_i | \Lambda}(x_i | \lambda)$$

$$\log\{f_{\Lambda | X_1, \dots, X_n}(\lambda | x_1, \dots, x_n)\} = \underline{\log(c)} + \underline{\log\{f_\Lambda(\lambda)\}} + \underline{\sum_{i=1}^{n} \log\{f_{X_i | \Lambda}(x_i | \lambda)\}}$$

As $n \to \infty$, $\frac{d}{d\lambda} \ell(\lambda | x_1, \dots, x_n)$ and $\frac{d^2}{d\lambda^2} \ell(\lambda | x_1, \dots, x_n)$

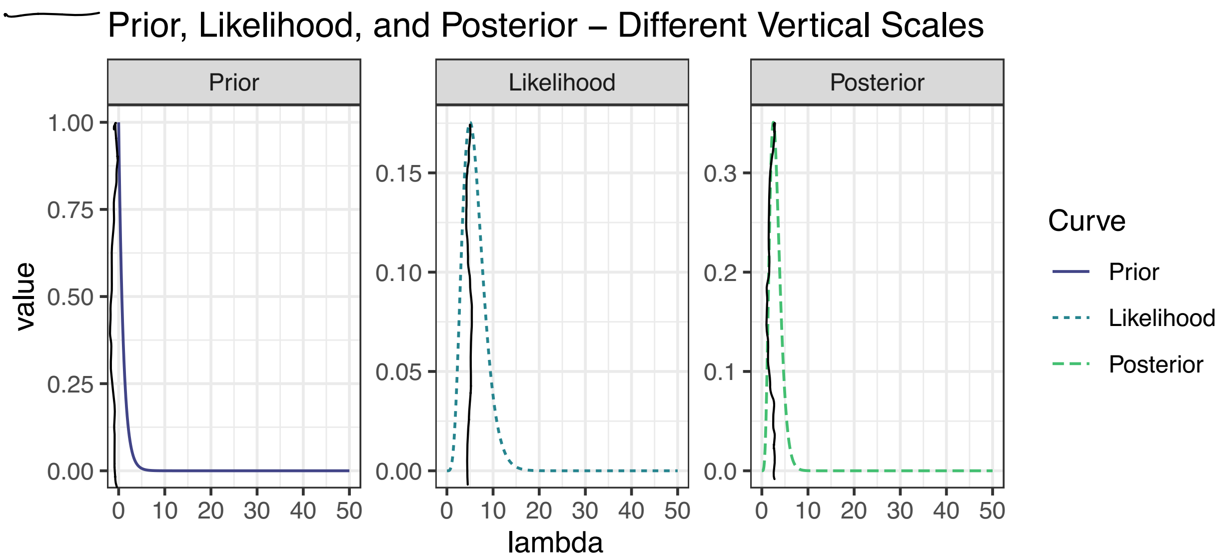
become more extreme

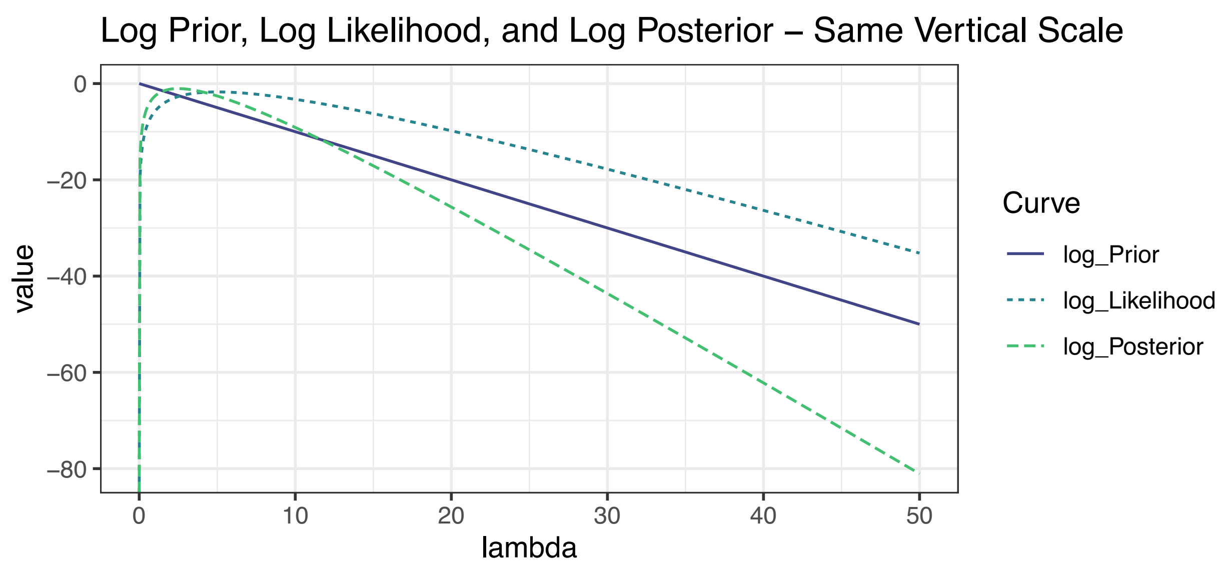
But prior derivatives stay the same

↳ prior looks constant for large n, relative to the log-likelihood.

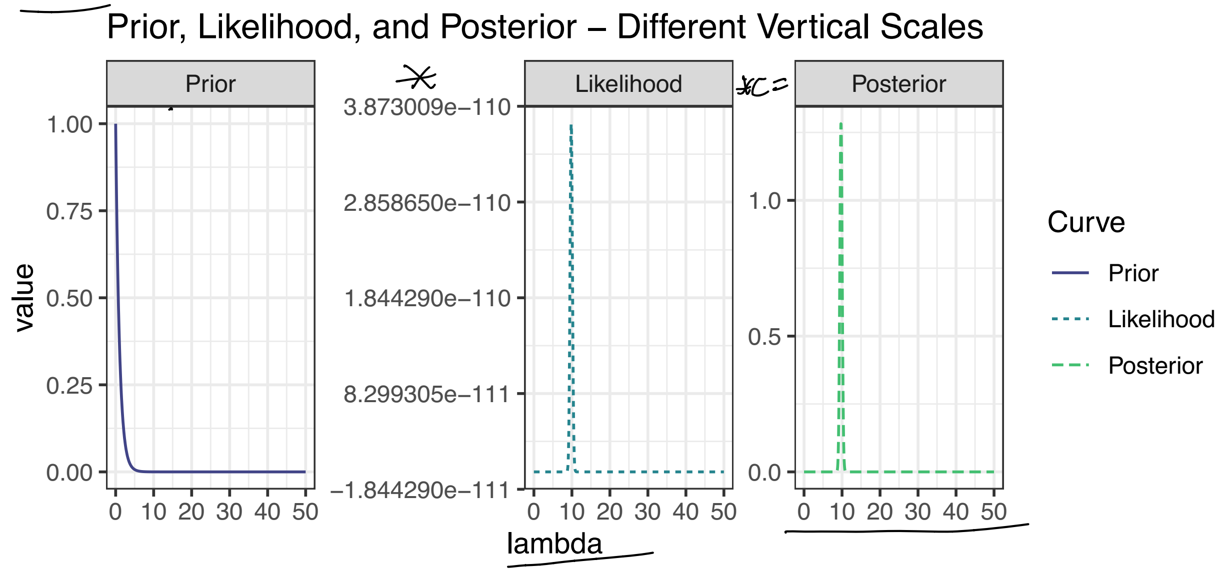
Simulation: Suppose $\lambda = 10$

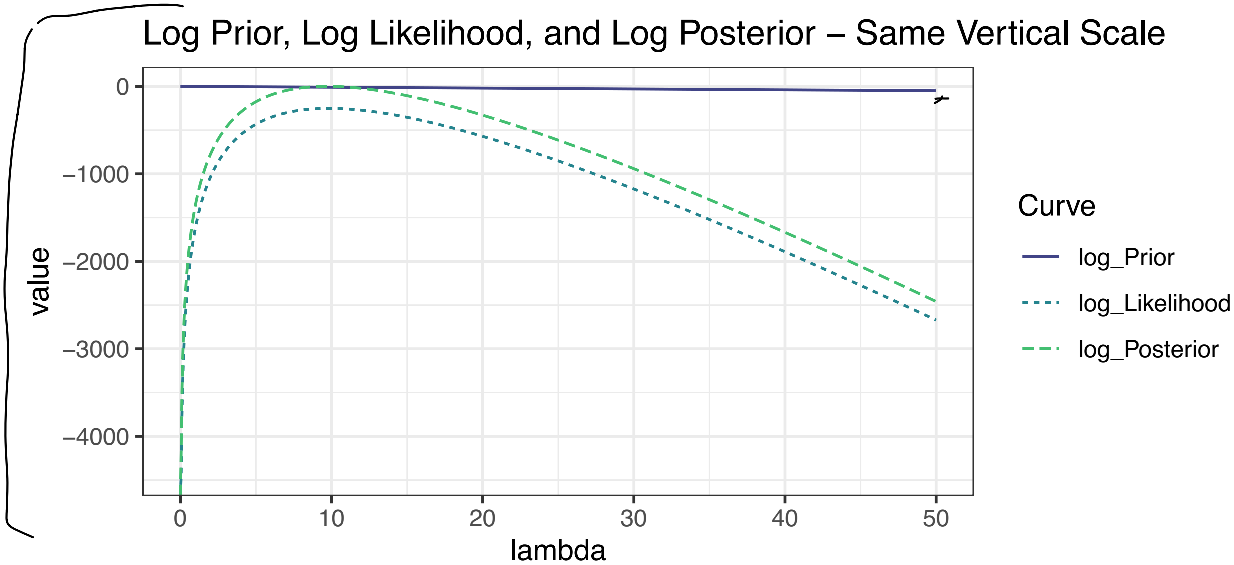
$\mathbf{n = 1}$





$n = 100$





for large n,
think of $\log\{f_\Lambda(\lambda)\}$ as approximately constant
in comparison to the log likelihood.