Stat 343 Practice with Monte Carlo Integration

Fish biology

This example is adapted from examples 10.4 and 11.6 in *Mathematical Statistics with Resampling and R* by Chihara and Hesterberg (2011). They write:

> "A marine biologist is investigating a species of trout in a certain area of California. She assumes that the lengths of these fish are normally distributed with mean μ (cm) and variance 8^2. She obtains a random sample of 15 fish and records their lengths $X_1, \ldots, X_{15} \sim \text{Normal}(\mu, 8^2)$. Based on her knowledge of this species at other locations, she [uses a prior distribution of] $\mu \sim \text{Normal}(50, 6^2)$. Suppose the mean of this random sample is $\bar{x} = 45$."

By plugging 8^2, 50, 6^2, and 45 into the formula we derived last class, it can be shown that the posterior distribution is $\mu|x_1, \ldots, x_{15} \sim \text{Normal}(45.53, 1.953^2)$

Chihara and Hesterberg continue:

> Biologists have determined that the relationship between the weight W and length X of a fish is apprroximately $W = aX^b$ for constant a and b that are determined empirically for any given species (Ricker (1973, 1975)). Suppose for this species, $W = 0.088 \times X^{3.069}$ g.

To summarize, we have the following:

Posterior distribution for μ based on our sample of size 15:

$\mu|x_1, \ldots, x_{15} \sim \text{Normal}(45.53, 1.953^2)$

For a new, as-yet unobserved fish from the population of this species of trout from California:

$X|\mu \sim \text{Normal}(\mu, 8^2)$

$W = 0.088 \times X^{3.069}$

(1) How could you find the posterior distribution of $X|x_1, \ldots, x_{15}$? Write down the set up as a suitable integral.

(2) Suppose you have available the pdf $f_{X|x_1,\ldots,x_{15}}(x|x_1, \ldots, x_{15})$ for the posterior distributon of $X|x_1, \ldots, x_{15}$. How could you find the expected value of $W|x_1, \ldots, x_{15}$? Write down the set up as a suitable integral.

(3) Suppose you have a sample from the posterior distribution of $X|x_1, \ldots, x_{15}$. How could you use this sample to approximately evaluate the integral you set up in part (2)?

(4) Suppose you have available the pdf $f_{X|x_1,\ldots,x_{15}}(x|x_1, \ldots, x_{15})$ for the posterior distributon of $X|x_1, \ldots, x_{15}$. How could you find the posterior probability that a newly sampled fish of this species in California will have a weight W between 25 and 30 pounds? Write down the set up as a suitable integral.

(5) Suppose you have a sample from the posterior distribution of $X|x_1, \ldots, x_{15}$. How could you use this sample to evaluate the integral you set up in part (4)?