$X_1, \ldots, X_n \overset{iid}{\sim} \text{Normal}(\mu, \sigma^2)$

$$f(x_i|\mu,\sigma) = (2\pi\sigma^2)^{-\frac{1}{2}} \exp\left\{\frac{-1}{2\sigma^2}(x_i-\mu)^2\right\}$$

$$f(x_1,\ldots,x_n|\mu,\sigma) = \prod_{i=1}^{n}\left[(2\pi\sigma^2)^{-1/2}\exp\left\{\frac{-1}{2\sigma^2}(x_i-\mu)^2\right\}\right]$$

$$= (2\pi\sigma^2)^{-n/2}\exp\left\{\frac{-1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right\}$$

$$\ell(\mu,\sigma|x_1,\ldots,x_n) = \log\left[(2\pi\sigma^2)^{-n/2}\right] - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$= \log\left[(2\pi)^{-n/2}\right] - n\log(\sigma) - \frac{1}{2}\sigma^{-2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{\partial}{\partial\mu}\ell(\mu,\sigma|x_1,\ldots,x_n) = \frac{-1}{2\sigma^2}\sum_{i=1}^{n}2(x_i-\mu)(-1)$$

$$= \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)$$

Set equal to 0:

$$0 = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)$$

$$\Rightarrow 0 = \sum_{i=1}^{n}x_i - n\mu$$

$$\Rightarrow \mu = \frac{1}{n}\sum_{i=1}^{n}x_i$$

(1) Estimator of μ is sample mean $\bar{X}$

$$\frac{\partial}{\partial\sigma}\ell(\mu,\sigma|x_1,\ldots,x_n) = \frac{-n}{\sigma} + \cancel{\frac{1}{2}}(\cancel{-2})\sigma^{-3}\sum_{i=1}^{n}(x_i-\mu)^2$$

Set equal to 0:

$$0 = \frac{-n}{\sigma} + \sigma^{-3}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\Rightarrow \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

Plug in critical value for μ:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

typical def. of s^2 has a denominator of $n-1$.