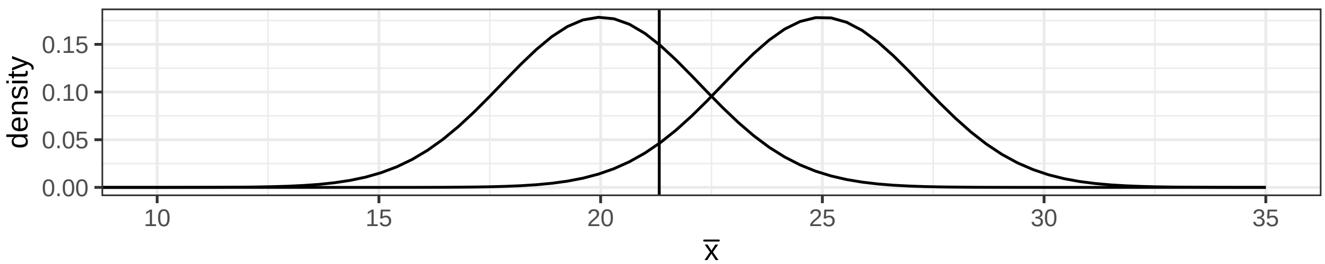
Warm Up: Power Functions for Hypothesis Tests

- Data Model: $X_1, \dots, X_5 \overset{\text{i.i.d.}}{\sim} \text{Normal}(\theta, 5^2)$
- We saw that the likelihood ratio test is *equivalent* to a test based on $\bar{x}$. The p-value is $P(\bar{X} \leq \bar{x} | \theta = 25)$ ("extreme" values of $\bar{x}$ are those that are at least as small as $\bar{x}$)

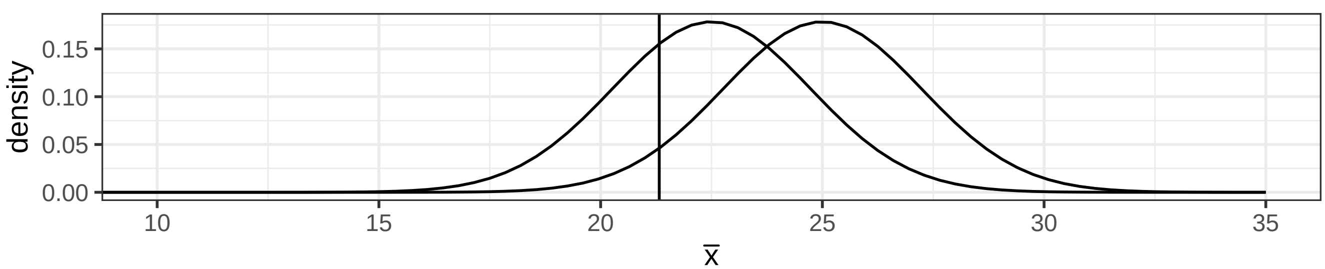
1. Consider a test of the hypotheses $H_0 : \theta = 25$ vs. $H_A : \theta = 20$. Below is a picture showing the pdf $f_{\bar{X}|\theta}(\bar{x}|20)$ of a Normal$(20, 5^2/5)$ distribution and the pdf $f_{\bar{X}|\theta}(\bar{x}|25)$ of a Normal$(25, 5^2/5)$ distribution, along with a vertical line at q_5^{null}, the 5th percentile of the Normal$(25, 5^2/5)$ distribution.

- **Shade in the area corresponding to $1 - \beta$, the power of the likelihood ratio test if H_A is correct.**
- **Show how you would calculate the power of the test as an integral of either $f_{\bar{X}|\theta}(\bar{x}|20)$ or $f_{\bar{X}|\theta}(\bar{x}|25)$**



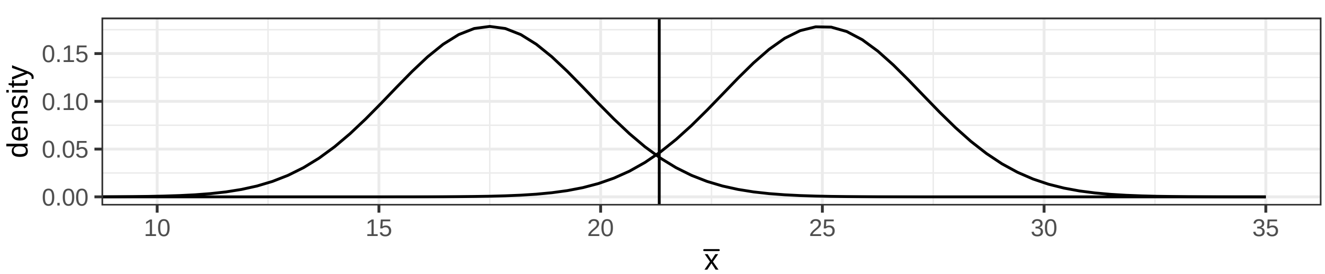
2. Suppose that instead we were testing the hypotheses $H_0 : \theta = 25$ vs. $H_A : \theta = 22.5$. Below is a picture showing the pdf $f_{\bar{X}|\theta}(\bar{x}|22.5)$ of a Normal$(22.5, 5^2/5)$ distribution and the pdf $f_{\bar{X}|\theta}(\bar{x}|25)$ of a Normal$(25, 5^2/5)$ distribution, along with a vertical line at q_5^{null}, the 5th percentile of the Normal$(25, 5^2/5)$ distribution.

- **Shade in the area corresponding to $1 - \beta$, the power of the likelihood ratio test if H_A is correct.**
- **Show how you would calculate the power of the test as an integral of either $f_{\bar{X}|\theta}(\bar{x}|22.5)$ or $f_{\bar{X}|\theta}(\bar{x}|25)$**



3. Suppose that instead we were testing the hypotheses $H_0 : \theta = 25$ vs. $H_A : \theta = 17.5$. Below is a picture showing the pdf $f_{\bar{X}|\theta}(\bar{x}|17.5)$ of a Normal$(17.5, 5^2/5)$ distribution and the pdf $f_{\bar{X}|\theta}(\bar{x}|25)$ of a Normal$(25, 5^2/5)$ distribution, along with a vertical line at q_5^{null}, the 5th percentile of the Normal$(25, 5^2/5)$ distribution.

- **Shade in the area corresponding to $1 - \beta$, the power of the likelihood ratio test if H_A is correct.**
- **Show how you would calculate the power of the test as an integral of either $f_{\bar{X}|\theta}(\bar{x}|17.5)$ or $f_{\bar{X}|\theta}(\bar{x}|25)$**



4. For which of the alternative hypotheses above ($\theta = 17.5$, $\theta = 20$, or $\theta = 22.5$) is the power of the test largest? For which is the power smallest?