- Let Ω be the parameter space of a model.
- We are wondering about the strength of evidence provided by the data against the "null hypothesis" that $\theta \in \Omega_0$ for some subset $\Omega_0 \subseteq \Omega$ of the parameter space.
- Identify a test statistic: $W = g(\underline{X})$ that can be used to summarize how consistent the data are with $H_0: \theta \in \Omega_0$.
  → want the dist'n to be different under H_0 and H_A.
- Find the sampling distribution of W if H_0 is true.
- Calculate the ~~p-value~~: value of the ~~test~~ statistic on our dataset:

  $$w = g(x_1, \dots, x_n)$$

- Calculate the p-value

informal: "$P(T$ more extreme than $t \mid H_0$ is true$)$"

~~More formal: $p = p(X_1, \dots, X_n)$ is a valid p-value if for any $\theta \in \Omega_0$ and any $\alpha \in [0,1]$~~

~~$P(p(X_1, \dots, X_n) \le \alpha) \le \alpha$~~

~~The probability of getting a~~

~~Slightly more formal: $\max_{\theta \in \Omega_0} P(W$ more extreme than $w \mid \theta)$~~

A small p-value means our data are not consistent with H_0.

~~As of~~

X = # blue M&M's   Obs. $x = 138$, out of $n = 541$

$X \sim \text{Binomial}(n, \theta)$

As of 2008, $\theta = 0.2$. Has it changed?

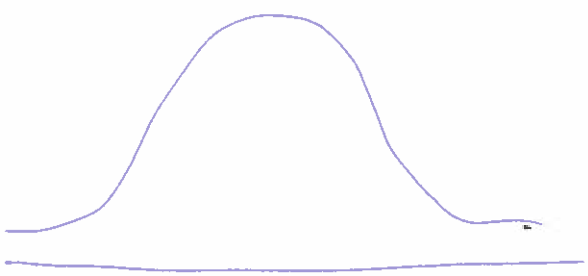
$\Omega = [0,1]$ $\Omega_0 = \{0.2\}$

$H_0: \theta \in \{0.2\}$

$H_1: \theta \notin \{0.2\}$

Test statistic: $W = X$

If H_0 is true, $W \sim \text{Binomial}(541, 0.2)$



$$W(X) = \frac{\mathcal{L}(\theta_0 | X)}{\mathcal{L}(\theta_1 | X)}$$

$$= \frac{f_X(X|\theta_0)}{f_X(X|\theta_1)}$$

$$= \frac{\cancel{\binom{n}{X}}\, \theta_0^{X} (1-\theta_0)^{n-X}}{\cancel{\binom{n}{X}}\, \theta_1^{X} (1-\theta_1)^{n-X}}$$

$$= \frac{0.2^{X}\, 0.8^{n-X}}{0.25^{X}\, 0.75^{n-X}}$$

<u>Ex:</u> We have 2 batches of paint, ...

$$f(\underset{\sim}{x}\mid\theta_0) = \left[\frac{1}{\sqrt{2\pi}\,5}\right]^5 \exp\left[\frac{-1}{2\cdot 25}\sum_{i=1}^{n}(x_i-25)^2\right]$$

$$f(\underset{\sim}{x}\mid\theta_1) = \left[\frac{1}{\sqrt{2\pi}\,5}\right]^5 \exp\left[\frac{-1}{2\cdot 25}\sum_{i=1}^{n}(x_i-10)^2\right]$$

$$W = \frac{f(\underset{\sim}{x}\mid\theta_0)}{f(\underset{\sim}{x}\mid\theta_1)} = \exp\left[\frac{-1}{2\cdot 25}\sum_{i=1}^{n}(x_i-25)^2 - \frac{-1}{2\cdot 25}\sum_{i=1}^{n}(x_i-10)^2\right]$$

we use

$$= \exp\left[\frac{-1}{50}\cdot\sum_{i=1}^{n}\left\{\cancel{x_i^2} - 50x_i + \underbrace{25^2}_{625} - \left(\cancel{x_i^2} - 20x_i + 10^2\right)\right\}\right]$$

$$= \exp\left[\frac{-1}{50}\sum_{i=1}^{n}(-30x_i + 525)\right]$$

$$= \exp\left[\frac{30}{50}\sum_{i=1}^{n}(x_i - 17.5)\right]$$

$$= \exp\left[\frac{3}{5}n(\bar{x} - 17.5)\right]$$

A smaller value of w is stronger evidence against H_0.

The smaller $\bar{x}$ is, the smaller w is.

$\therefore$ we can calculate our p-value by looking at $\bar{X}$.

If H_0 is true, $\bar{X} \sim \text{Normal}\left(25, \frac{5^2}{5}\right)$