Large-Sample Normal Approximation to the Posterior

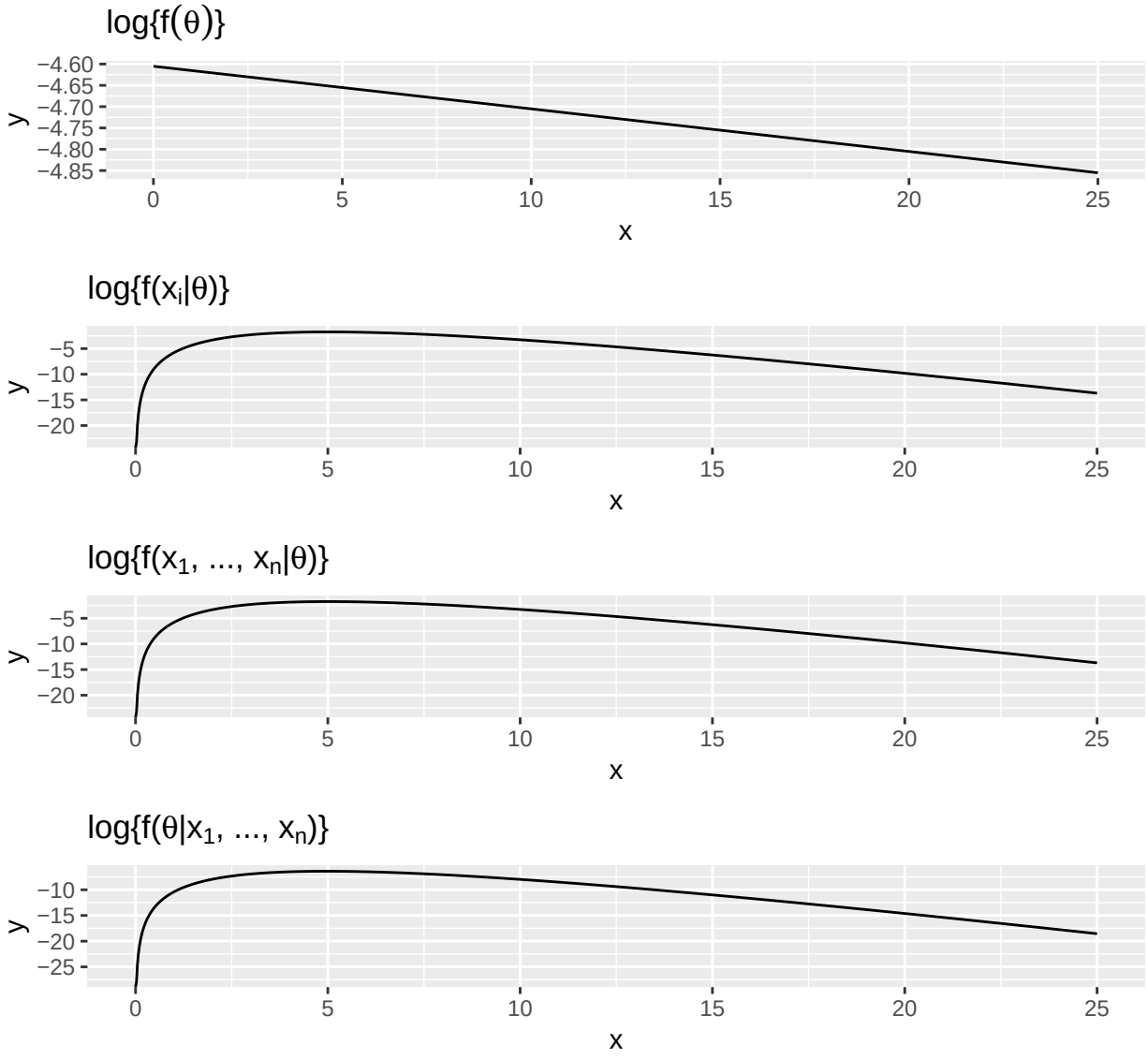
Poisson Model

- Observe $X_1, \ldots, X_n$; X_i is the number of seedlings in quadrat number i.
- Data Model: $X_i|\Lambda = \lambda \overset{\text{i.i.d.}}{\sim} \text{Poisson}(\lambda)$
- Suppose we use a Gamma prior for Λ
  - Example: $\Lambda \sim \text{Gamma}(1, 0.01)$ is fairly non-informative
- Decomposing the log-posterior pdf into contributions from prior and likelihood

$$\log\{f_{\Theta|X_1,\ldots,X_n}\} = \log\{c \cdot f_\Theta(\theta) \cdot \prod_{i=1}^n f_{X_i|\Theta}(x_i|\theta)\} = \log(c) + \log\{f_\Theta(\theta)\} + \sum_{i=1}^n \log\{f_{X_i|\Theta}(x_i|\theta)\}$$

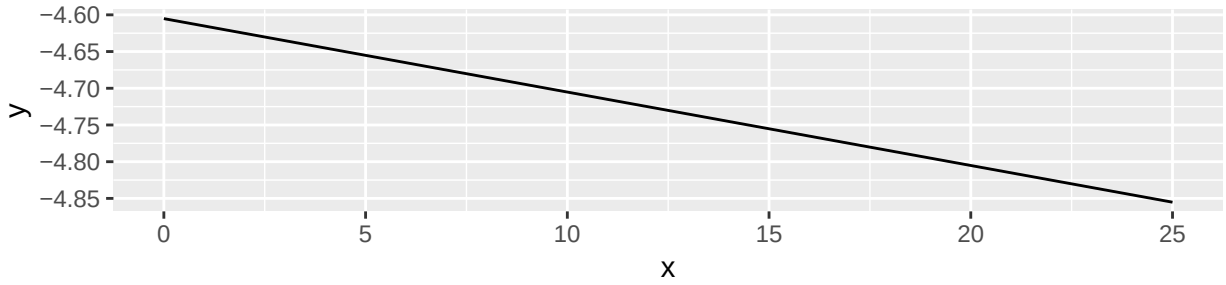
Simulation: $\lambda = 10$

n = 1

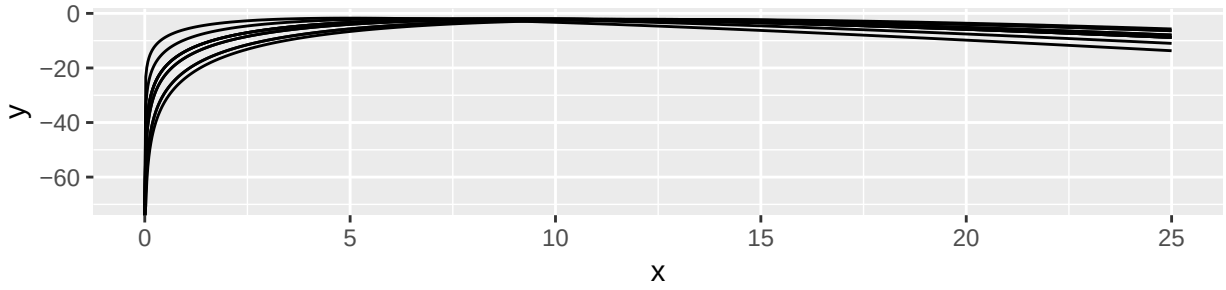


n = 10

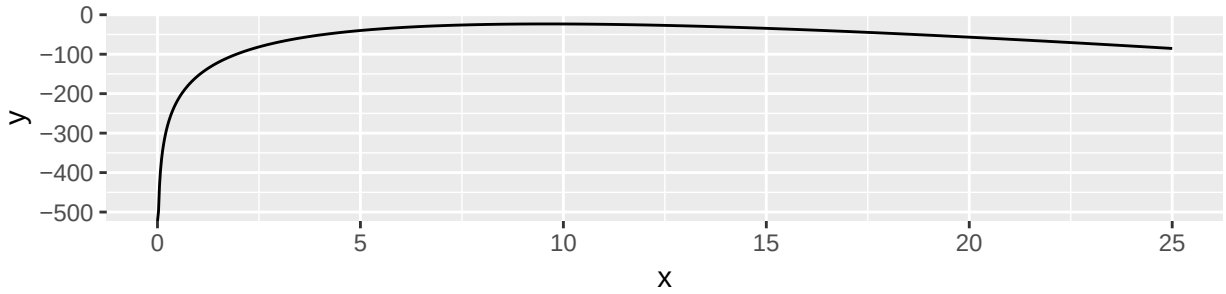
log{f(θ)}



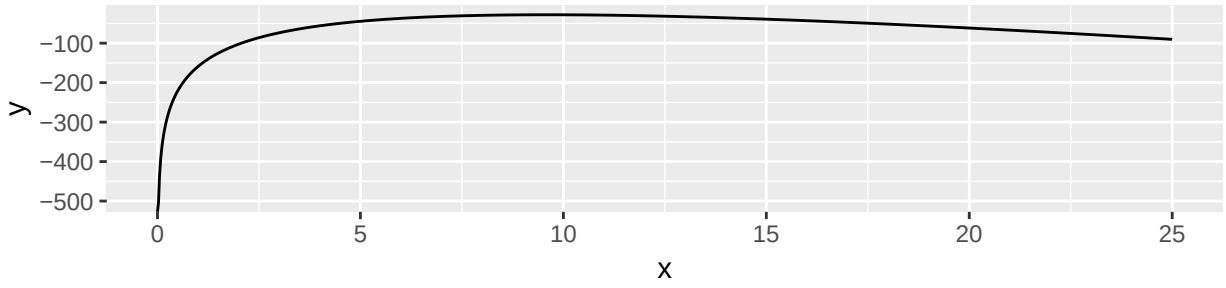
log{f(x_i|θ)}



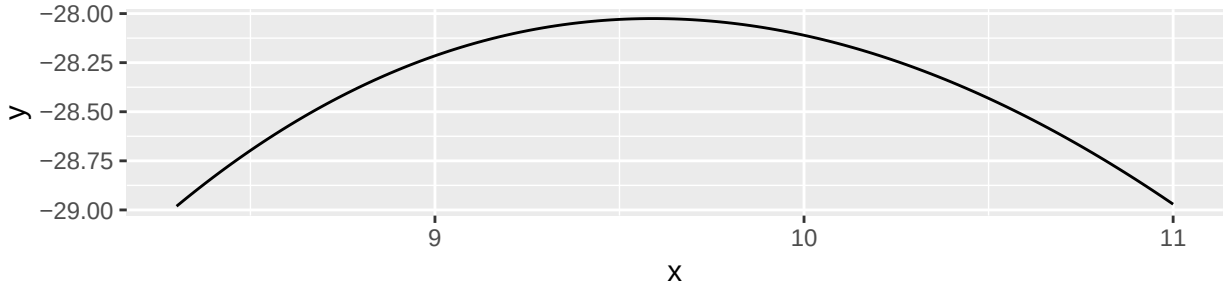
log{f(x_1, ..., x_n|θ)}



log{f(θ|x_1, ..., x_n)}

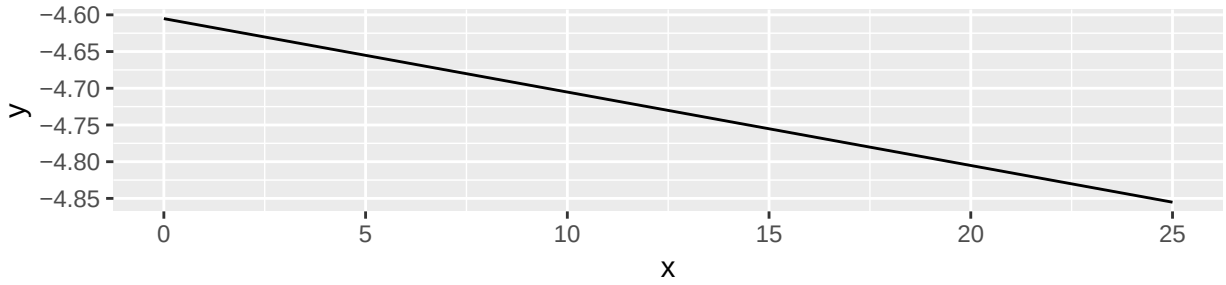


log{f(θ|x_1, ..., x_n)}, zoomed in to [8.3, 11.0], width 2.7

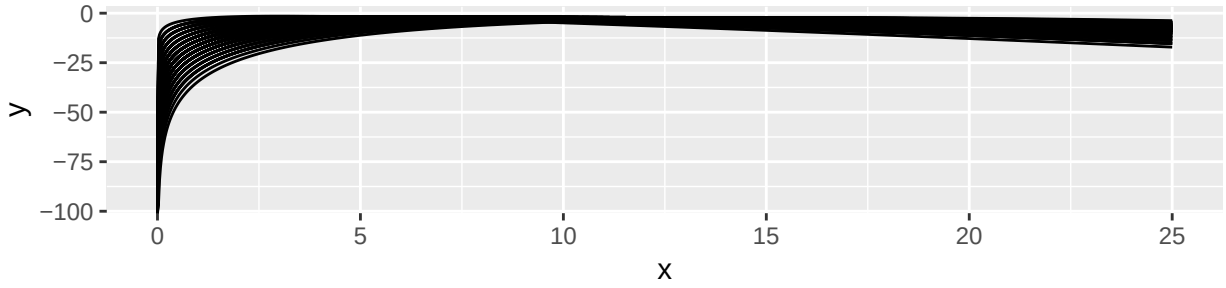


n = 100

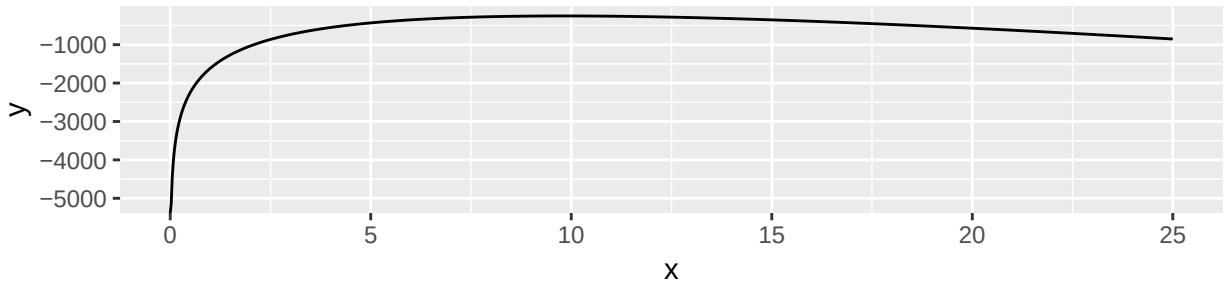
log{f(θ)}



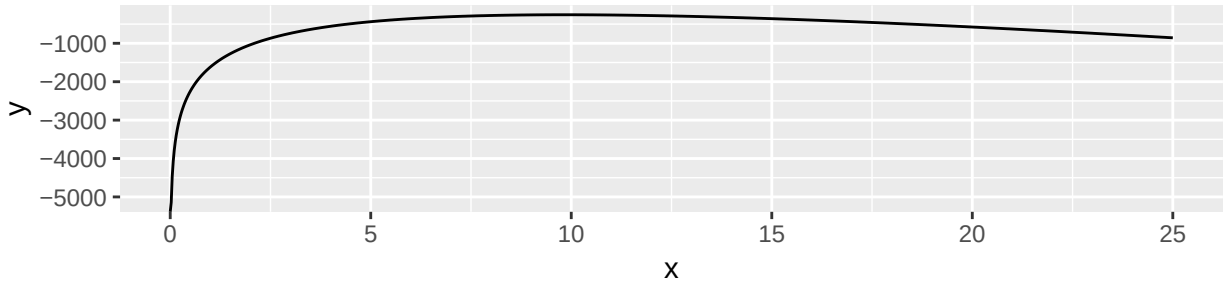
log{f(x_i|θ)}



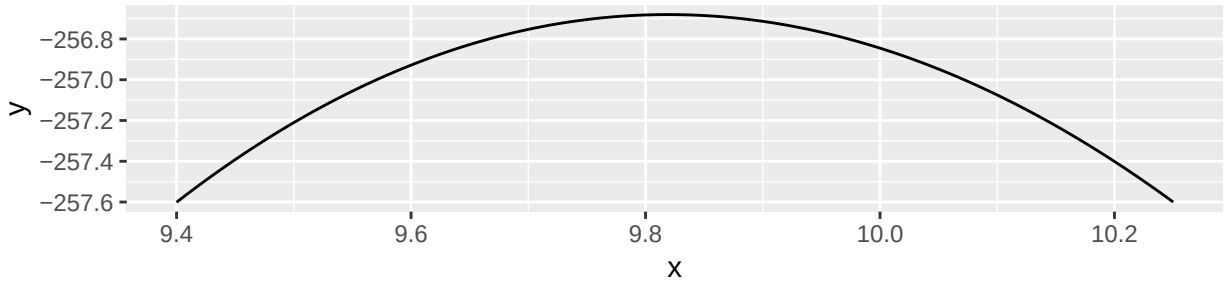
log{f(x_1, ..., x_n|θ)}



log{f(θ|x_1, ..., x_n)}

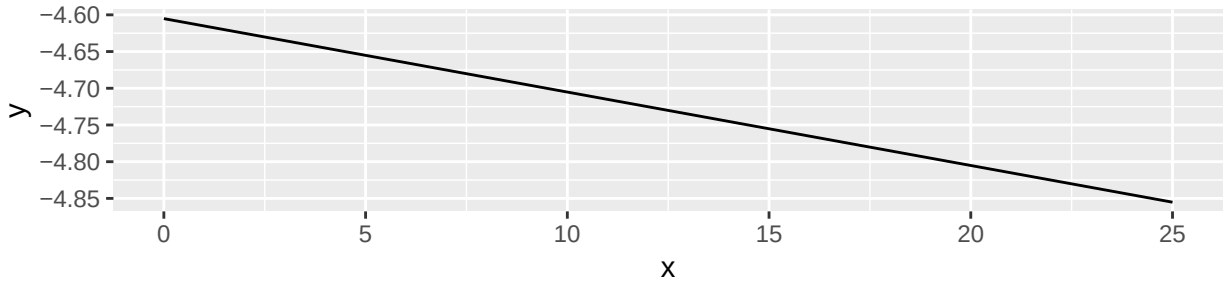


log{f(θ|x_1, ..., x_n)}, zoomed in to [9.4, 10.25], width 0.85

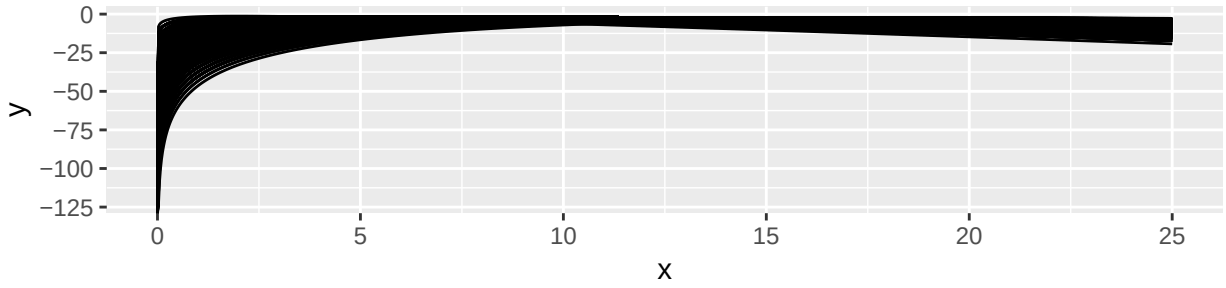


n = 1000

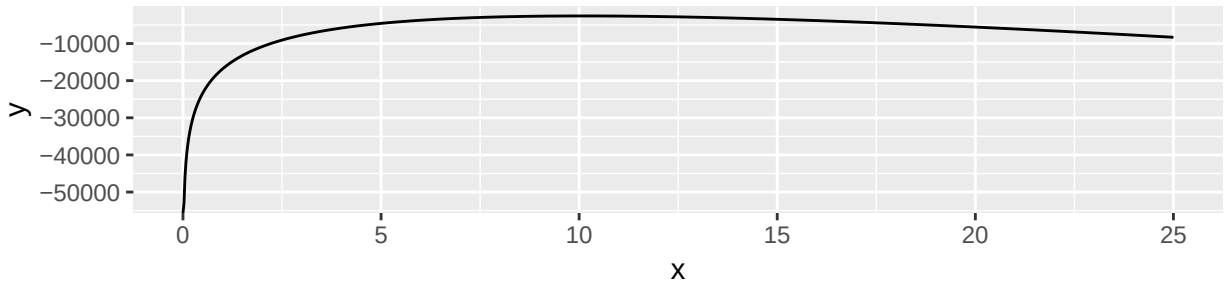
log{f(θ)}



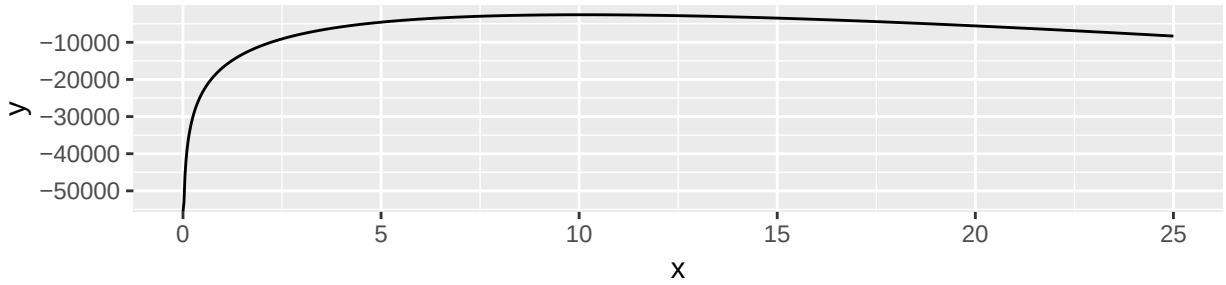
log{f(x_i|θ)}



log{f(x_1, ..., x_n|θ)}



log{f(θ|x_1, ..., x_n)}



log{f(θ|x_1, ..., x_n)}, zoomed in to [9.95, 10.25], width 0.3