

$$\text{MSE}(\hat{\theta}) = E[(\hat{\theta} - \theta)^2]$$

$$= E[\{\hat{\theta} - E(\hat{\theta}) + E(\hat{\theta}) - \theta\}^2]$$

$$= E[\{\hat{\theta} - E(\hat{\theta})\}^2 + 2\{\hat{\theta} - E(\hat{\theta})\}\{E(\hat{\theta}) - \theta\} + \{E(\hat{\theta}) - \theta\}^2]$$

$$= E[\{\hat{\theta} - E(\hat{\theta})\}^2] + 2\{E(\hat{\theta}) - \theta\}E[\hat{\theta} - E(\hat{\theta})] + \{E(\hat{\theta}) - \theta\}^2$$

$$= \text{Var}(\hat{\theta}) + 2\{E(\hat{\theta}) - \theta\}\{E(\hat{\theta}) - E(\hat{\theta})\} + \{\text{Bias}(\hat{\theta})\}^2$$

$$= \text{Var}(\hat{\theta}) + \{\text{Bias}(\hat{\theta})\}^2$$