Example of fitted values and orthogonal projections

This example will be due as part of your next homework assignment. We're just getting a start on it in class together.

Model Statement

Suppose we use the model

$$y_i = \beta + \varepsilon_i$$
$$\varepsilon_i \sim \text{Normal}(0, \sigma^2)$$

Also suppose we have $n = 2$ observations, and the observed response vector is $y = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(a) What is the design matrix X?

(b) Find the hat matrix H.

(c) Find the fitted values $\hat{y} = Hy$.

(d) Draw a figure showing $\mathcal{C}(X)$ (it is a line), y, and $\hat{y}$, clearly labelling each. Connect y and $\hat{y}$ with a line segment, and by drawing an appropriate right angle on your figure, illustrate that $\hat{y}$ is the orthogonal projection of y onto $\mathcal{C}(X)$.