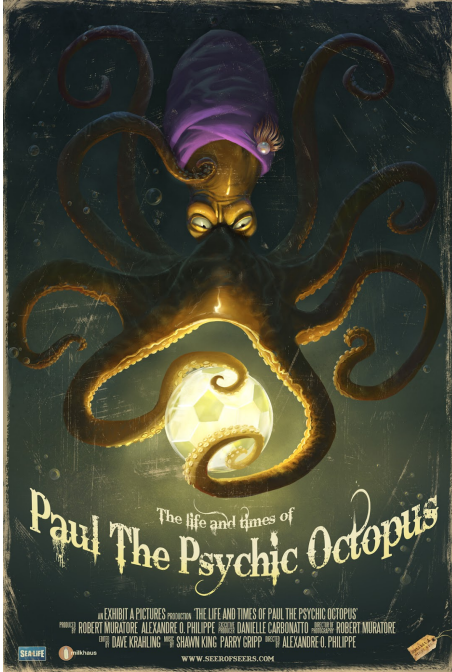


Hypothesis Tests about Proportions

1. Identify the
 - **population parameter**: a number summarizing the population.
 - What proportion of the “population” are in a certain category?
 - Denote by p (for proportion).
 - **sample statistic**: a number summarizing the sample.
 - In our sample, how many observational units are in that category?
 - Denote by X for a random variable, x for the observed value
2. State our hypotheses.
 - **Null hypothesis** (H_0): Nothing interesting is going on.
 - State in words what this means for the example we’re looking at.
 - State as an equality involving the population parameter: $p = \underline{\hspace{1cm}}$
 - **Alternative hypothesis** (H_A): Something interesting is going on.
 - State in words what this means for the example we’re looking at.
 - State as an inequality involving the population parameter: $p < \underline{\hspace{1cm}}$, or $p > \underline{\hspace{1cm}}$, or $p \neq \underline{\hspace{1cm}}$.
3. Find the sampling distribution for the sample statistic, assuming H_0 is true.
 - For now, our only choice is $\mathbf{X} \sim \mathbf{Binomial}(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$ (fill in sample size and the value of p from the null hypothesis)
 - We need to **check conditions** before proceeding:
 - Were there any forms of bias in our sampling procedure?
 - Was a categorical variable recorded for each observational unit?
 - Is our sample statistic a count of how many observational units in our sample were in one of the categories?
 - Are different observational units in our sample independent?
4. Calculate a p-value: the probability of getting a test statistic at least as extreme as what we observed, assuming H_0 is true.
5. Draw a conclusion. Two options:
 - Make a statement about the strength of evidence against the null hypothesis provided by the sample data.
 - A p-value of .10 or less: some evidence against H_0
 - A p-value of .05 or less: fairly strong evidence against H_0
 - A p-value of .01 or less: very strong evidence against H_0
 - A p-value of .001 or less: extremely strong evidence against H_0
 - Make a decision by comparing the p-value to a specified significance level α .
 - Most commonly used values are $\alpha = 0.05$ and $\alpha = 0.01$, but there is no specific justification for these values.
 - “Reject” H_0 if the p-value is less than α
 - “Fail to Reject” H_0 if the p-value is greater than or equal to α

Example 1: Is Paul the Octopus Psychic?



(image credit: Paul J. Sullivan)

We observed 8 successful predictions out of 8 attempts.

Step 1: Identify population parameter and sample statistic.

- Population parameter:
- Sample statistic:

Step 2: State hypotheses

- Null Hypothesis (H_0):
- Alternative Hypothesis (H_A):

Step 3: Find sampling distribution of sample statistic, assuming H_0 is true

Check conditions for using a Binomial distribution:

- Were there any forms of bias in our sampling procedure?
- Was a categorical variable recorded for each observational unit?
- Is our sample statistic a count of how many observational units in our sample were in one of the categories?
- Are different observational units in our sample independent?

State the sampling distribution, assuming H_0 is true:

$X \sim \text{Binomial}(\text{---}, \text{---})$

Step 4: Calculate the p-value for the test

The p-value is the probability of getting a test statistic at least as extreme as what we observed, assuming H_0 is true.

In this example, that means $P(X \geq 8) = P(X > 7)$.

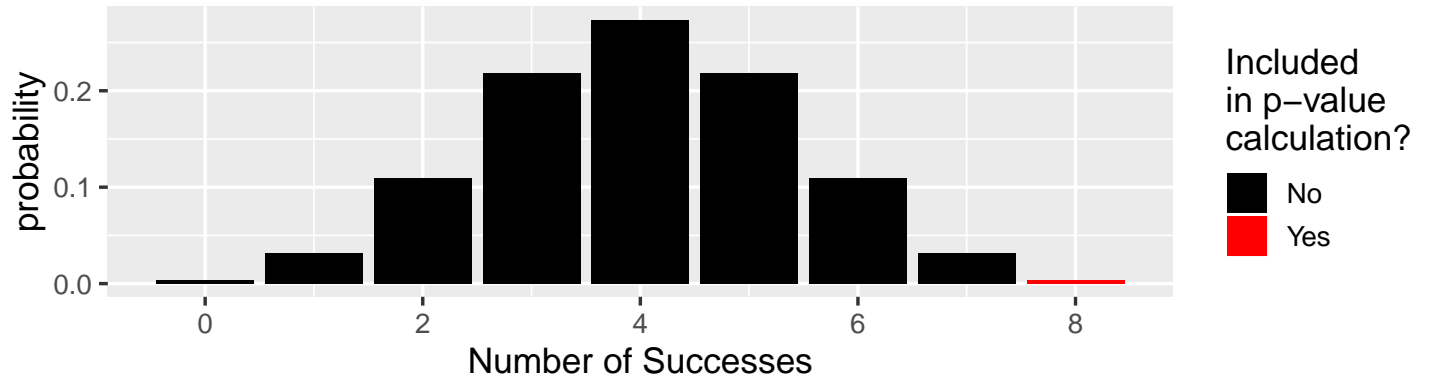
```
pbinom(q = 7, size = 8, prob = 0.5, lower.tail = FALSE)
```

```
## [1] 0.00390625
```

```
binom.test(x = 8,  
  n = 8,  
  p = 0.5,  
  alternative = "greater")
```

```
##  
##  
##  
## data: 8 out of 8  
## number of successes = 8, number of trials = 8, p-value = 0.003906  
## alternative hypothesis: true probability of success is greater than 0.5
```

```
## 95 percent confidence interval:  
## 0.687656 1.000000  
## sample estimates:  
## probability of success  
## 1
```



Step 5: Draw a conclusion

What is the strength of evidence against the null hypothesis provided by the data?

Make a decision using a significance level of $\alpha = 0.01$.

Example 2: Blue M&M's

My favorite color of M&M's is blue. Mars Company (the makers of M&M's) last published the distribution of colors for M&M's in 2008, and at that time the proportion of M&M's that were blue was 0.16. However, I'm concerned that they may be putting fewer blue M&M's in now. I take a sample of 100 M&M's and count 12 blue M&M's in my sample. Does this provide evidence that the proportion of M&M's that are blue is lower now than it was in 2008?

Step 1: Identify population parameter and sample statistic.

- Population parameter:

- Sample statistic:

Step 2: State hypotheses

- Null Hypothesis (H_0):

- Alternative Hypothesis (H_A):

Step 3: Find sampling distribution of sample statistic, assuming H_0 is true

Check conditions for using a Binomial distribution:

- Were there any forms of bias in our sampling procedure?
- Was a categorical variable recorded for each observational unit?
- Is our sample statistic a count of how many observational units in our sample were in one of the categories?
- Are different observational units in our sample independent?

State the sampling distribution, assuming H_0 is true:

$X \sim \text{Binomial}(\text{---}, \text{---})$

Step 4: Calculate the p-value for the test

The p-value is the probability of getting a test statistic at least as extreme as what we observed, assuming H_0 is true.

In this example, that means $P(X \leq 12)$

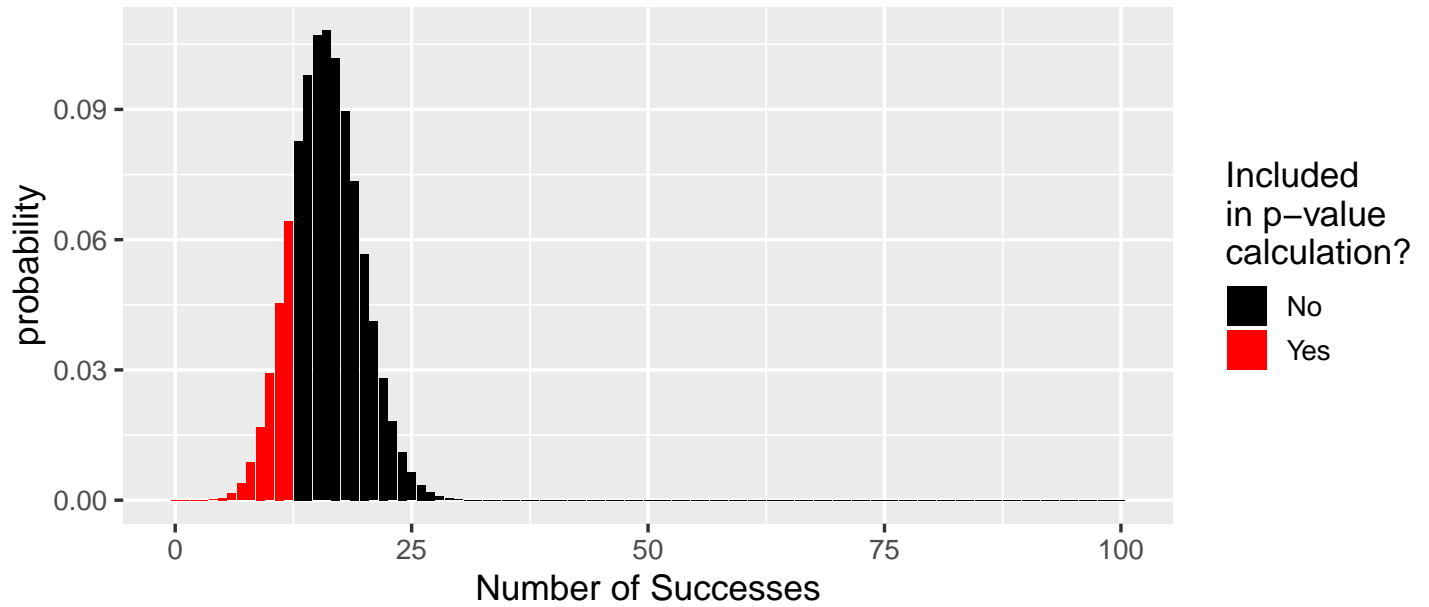
```
pbinom(q = 12, size = 100, prob = 0.16)
```

```
## [1] 0.1703155
```

```
binom.test(x = 12,  
  n = 100,  
  p = 0.16,  
  alternative = "less")
```

```
##  
##  
##  
## data: 12 out of 100  
## number of successes = 12, number of trials = 100, p-value = 0.1703  
## alternative hypothesis: true probability of success is less than 0.16
```

```
## 95 percent confidence interval:  
## 0.0000000 0.1871661  
## sample estimates:  
## probability of success  
## 0.12
```



Step 5: Draw a conclusion

What is the strength of evidence against the null hypothesis provided by the data?

Make a decision using a significance level of $\alpha = 0.05$.

Example 3: Absenteeism

According to the National Center for Education Statistics, in 1996, 66% of grade-school students in the U.S. had missed at least one day of school in the past month. A more recent survey of 8302 randomly selected students found that 5562 of them had missed at least one day of school in the past month. Has the rate of absenteeism changed since 1996?

Step 1: Identify population parameter and sample statistic.

- **Population parameter:**

- **Sample statistic:**

Step 2: State hypotheses

- **Null Hypothesis (H_0):**

- **Alternative Hypothesis (H_A):**

Step 3: Find sampling distribution of sample statistic, assuming H_0 is true

Check conditions for using a Binomial distribution:

- Were there any forms of bias in our sampling procedure?
- Was a categorical variable recorded for each observational unit?
- Is our sample statistic a count of how many observational units in our sample were in one of the categories?
- Are different observational units in our sample independent?

State the sampling distribution, assuming H_0 is true:

$X \sim \text{Binomial}(\text{---}, \text{---})$

Step 4: Calculate the p-value for the test (we won't do this manually for 2-sided tests about a proportion)

The p-value is the probability of getting a test statistic at least as extreme as what we observed, assuming H_0 is true.

In this example, that means “at least as far from the expected number of absentees if H_0 was true”.

Expected number of absentees if H_0 is true: $8302 * 0.66 \approx 5479$

Our observed value was 5562.

How far above the expected value is this? $5562 - 5479 = 83$

“As extreme” in the other direction would be... $5479 - 83 = 5396$

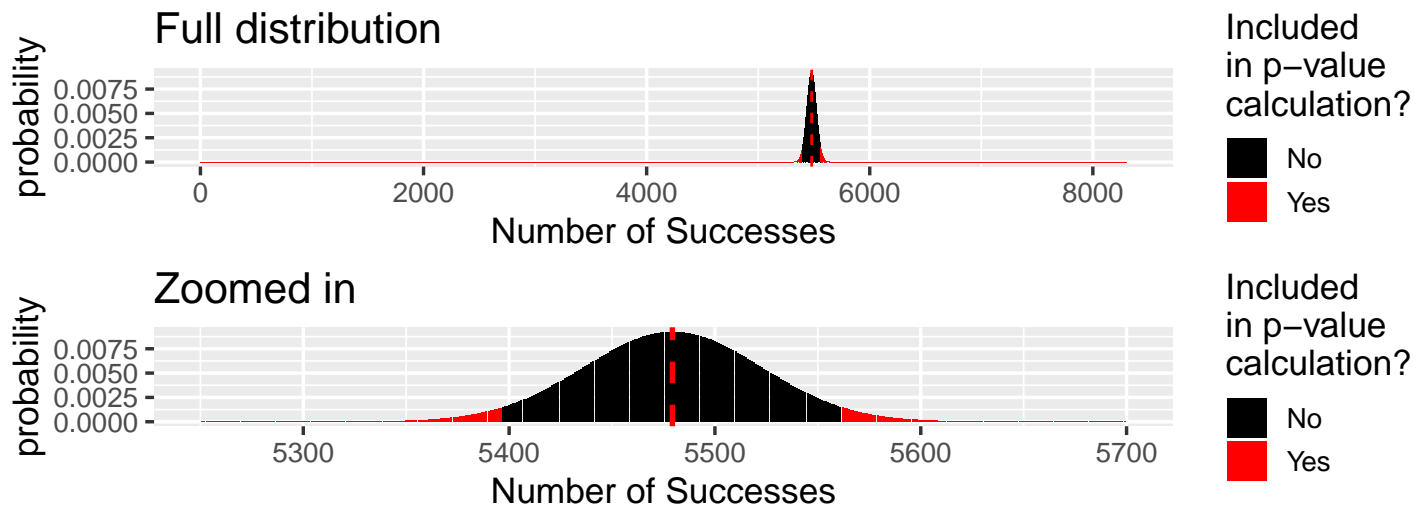
...so the p-value is: $P(X \leq 5396) + P(X \geq 5562)$

```
pbinom(5396, size = 8302, prob = 0.66) +  
  pbinom(5562 - 1, size = 8302, prob = 0.66, lower.tail = FALSE)
```

```
## [1] 0.05595045
```

```
binom.test(x = 5562,
  n = 8302,
  p = 0.66,
  alternative = "two.sided")
```

```
##
##
##
## data: 5562 out of 8302
## number of successes = 5562, number of trials = 8302, p-value =
## 0.05595
## alternative hypothesis: true probability of success is not equal to 0.66
## 95 percent confidence interval:
## 0.6597255 0.6800731
## sample estimates:
## probability of success
## 0.669959
```



Step 5: Draw a conclusion

What is the strength of evidence against the null hypothesis provided by the data?

Make a decision using a significance level of $\alpha = 0.05$.